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Survival conditions of a vapour bubble in saturated liquid flowing inside a micro-channel

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Abstract

The conditions dictated by a vapour bubble to exist in saturated liquid flowing inside a micro-channel are the main subject of the paper. The liquid flow is laminar and fully established at a constant wall heat flux. Under these conditions, an equilibrium bubble is shown to require a heat flux, which sensitively increases with decreasing channel radius. Given a mass flow density, the hydrodynamic forces acting on the bubble generated in a macro-channel may cause its premature detachment thereby shifting formation of "visible" bubbles towards higher heat fluxes in comparison to macro-channels.

1. Introduction

Modelling of bubble nucleation in liquids requires the state of the mother phase to be specified, in the first place, and, in the case of a heterogeneous nucleation, the interaction of this phase with its surroundings. This implies that the governing field equations for the single phase have to be solved simultaneously. Difficulties associated with a treatment of these equations under flow conditions of the parent phase are usually circumvented by prescribing particular states, whereas the interaction effects are mostly accounted for in terms of interfacial tensions. Along such a way of treatment, one arrives at relationships which state the conditions for a vapour bubble to survive with pool boiling or flow boiling, see e.g. [1–11]. As a rule, the flow is considered to be unconfined, that is to say, the equilibrium bubble is much smaller than the linear measures of the flow field. Recently, Peng et al. [12] provided an expression, which should be valid under conditions of flow boiling in a narrow (micro) channel where the bubble diameter is comparable with the channel diameter. The structure of this equation strongly differs from the ones valid for flows in macro-channels.

In the present paper, we propose another expression for the heat flux necessary for a vapour bubble, if once formed, to be able to grow under micro-channel flow conditions. The following assumptions are adopted:

- thermally and hydrodynamically developed laminar flows;
- single component fluid of constant physical properties;
- constant wall heat flux;
- circular cross-sectional flow area of the channel.

The temperature distribution $T(r,x)$ in a flow thus specified is known, see e.g. [13], to be

$$T - T_{\mathrm{W}} = 2\frac{\rho_{\mathrm{L}} c_{\mathrm{pL}} u R^2}{k_{\mathrm{L}}}\left(\frac{1}{4}\left(\frac{r}{R}\right)^2 - \frac{1}{16}\left(\frac{r}{R}\right)^4 - \frac{3}{16}\right)\frac{\partial T}{\partial x}, \tag{1}$$

where u is the average fluid velocity and $\partial T/\partial x$ is the temperature derivative in the flow direction x; $k_{\mathrm{L}}, \rho_{\mathrm{L}}$ and c_{pL} denote common physical properties, whereas the meaning of r, R and T_{W} follows from Fig. 1(a).

The basic line of the derivation procedure taken for the present considerations is the same as used, e.g., in [3,4,7] for macro-channel flows; the reader may be referred to these sources for details. Here, it should suffice to note the main assumption of the model, namely, the bubble generation does not affect the temperature field in the liquid.

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Nomenclature

A	surface area
a	thermal diffusivity
c	specific heat capacity, constant
F	force
Δh	enthalpy difference (latent heat)
k	thermal conductivity
$\dot{m}$	mass flow density
p	pressure
q	heat flux
R	channel radius, bubble radius
r	radial coordinate, bubble radius
S	entropy
T	temperature
t	time
ΔT	temperature difference
u	average (local in Eqs. (17) and (18)) axial velocity
V	volume
x	axial coordinate
ν	kinematic viscosity
ρ	density
σ	surface tension
τ	shear stress

Indices

B	bubble
L	liquid
LV	liquid–vapour
S	constant entropy
T	constant temperature
V	vapour
W	wall
∞	plane interface

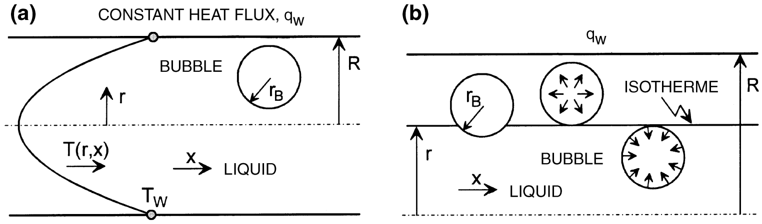

Fig. 1. Spherical vapour bubble in a liquid flowing inside a micro-channel heated at a constant heat flux. (a) General illustration of the model. (b) Sketch showing a vapour bubble at different radial positions, but in the same axial position. Strong hydrodynamic forces in micro-channels may lead to smaller bubble detachment sizes. When a detached bubble is driven towards the channel axis it might become unstable with respect to its new surroundings and condense. The arrows indicate the moving direction of the bubble surface.

2. Relationship between heat flux and bubble size

Whatever processes may generate a vapour bubble, see e.g. [14–18], particular conditions must be met for the bubble to survive. These conditions are dictated by the bubble equilibrium. As it is well known, e.g. [3], this equilibrium may be expressed for a pure homogeneous liquid embedding a spherical vapour bubble in terms of temperature as

$$T_B - T_\infty = 2\frac{\sigma T_\infty}{h_{LV}\rho_V r_B}, \tag{2}$$

where T_∞ and T_B are, respectively, the equilibrium temperatures at the plane interface and the bubble surface having the curvature radius r_B. This equation ignores the effects of external fields and wall proximity, which, each for itself, may distort the spherical shape of the equilibrium bubble [19]. However, for simplicity, Eq. (2) is considered to be sufficiently accurate for the present purposes.

Under conditions of heat transfer, the temperature of the liquid is inhomogeneous and the interface of a bubble generated on the heated channel wall is probably not isothermal. At portions of the interface near the wall surface, a higher interface temperature is expected and evaporation will take place there, while condensation – even under overall saturation conditions – may occur in the wall-far region of the interface. In such a case, the vapour bubble acts as a heat pipe; it will grow when evaporation overcomes condensation.

The equilibrium at the surface of the bubble illustrated in Fig. 1(a) may be assumed to occur at the places where the temperatures T and T_B in Eqs. (1) and (2) take the same values, $T = T_B$. Thus, considering the expression

$$\frac{\partial T}{\partial x} = 2\frac{q_W}{\rho_L c_{pL} u R}, \tag{3}$$

valid at constant wall heat flux q_W under the conditions of Eq. (1), and furthermore, setting $T_W - T_\infty = \Delta T$, one obtains from Eqs. (1) and (2)

$$\left(\left(\frac{3}{4}-\left(\frac{r}{R}\right)^2+\frac{1}{4}\left(\frac{r}{R}\right)^4\right)R-\frac{k_L\Delta T}{q_W}\right)\frac{r_B}{R}R+2\times\frac{\sigma T_\infty k_L}{h_{LV}\rho_V q_W}=0. \tag{4}$$

The radius r in this equation measures the distance of the points of the bubble surface, where the equilibrium occurs, from the channel axis. Analytically, this is the intersection line of the bubble surface and the isothermal cylindrical surface ($T = T_B$) having the radius r, Fig. 1(b). A bubble, touching this isothermal surface from outside, is completely surrounded by liquid of a sufficient superheat and will grow. On the contrary, the same bubble, touching this surface from inside, is not able to survive.

The radii r_B and R can be linked with each other by specifying the radial bubble position in the channel. For instance, if the bubble, having a diameter less than the channel radius, is adhering to the wall surface and the equilibrium is taken to occur on the bubble vertex (wall distance $2r_B$), then, $r_B/R = (1-(r/R))/2$, and Eq. (4) gives

$$\left(\left(\frac{3}{4}-\left(\frac{r}{R}\right)^2+\frac{1}{4}\left(\frac{r}{R}\right)^4\right)R-\frac{k_L\Delta T}{q_W}\right)\left(1-\frac{r}{R}\right)\frac{R}{2}+2\times\frac{\sigma T_\infty k_L}{h_{LV}\rho_V q_W}=0. \tag{5}$$

The simplest way to treat this equation further is to choose the radial position r; by this, one specifies the bubble size. For example, with $r \to R$ the bubble radius tends to zero, whereas for $r = 0$ one postulates a vapour bubble which extends from the wall surface up to the channel axis ($r_B/R = 1/2$), and the equation becomes

$$R^2-\frac{8}{3}\frac{k_L\Delta T}{q_W}R+\frac{16}{3}\frac{\sigma T_\infty k_L}{h_{LV}\rho_V q_W}=0. \tag{6}$$

If the bubble occupies the whole cross-sectional flow area ($r_B = R$) and if an equilibrium is assumed to occur in the channel axis ($r = 0$), Eq. (4) delivers [1]

$$R^2-\frac{4}{3}\frac{k_L\Delta T}{q_W}R+\frac{8}{3}\frac{\sigma T_\infty k_L}{h_{LV}\rho_V q_W}=0 \tag{7}$$

or

$$R-\frac{2}{3}\frac{k_L\Delta T}{q_W}=\pm\frac{2}{3}\frac{k_L\Delta T}{q_W}\times\left(1-6\frac{\sigma T_\infty k_L}{h_{LV}\rho_V q_W}\left(\frac{q_W}{k_L\Delta T}\right)^2\right)^{1/2}. \tag{8}$$

To get a unique solution for the channel radius R, the term in the square root brackets must be zero, thus,

$$1-6\frac{\sigma T_\infty k_L}{h_{LV}\rho_V q_W}\left(\frac{q_W}{k_L\Delta T}\right)^2=0. \tag{9}$$

Hence

$$R-\frac{2}{3}\frac{k_L\Delta T}{q_W}=0 \tag{10}$$

or

$$q_W-\frac{1}{6}\frac{h_{LV}\rho_V k_L}{\sigma T_\infty}(\Delta T)^2=0 \tag{9a}$$

and

$$q_W=\frac{2}{3}\frac{k_L}{R}\Delta T=\frac{2}{3}\frac{k_L}{R}(T_W-T_\infty). \tag{10a}$$

Combining the latter two gives

$$q_W=\frac{4}{3}\frac{2\sigma T_\infty}{h_{LV}\rho_V R}\frac{k_L}{R}. \tag{11}$$

To interpret this equation, we might see the term $(4/3)k_L/R$ to be a heat transfer coefficient. The heat flux is then calculated by multiplying this heat transfer coefficient with the rise of the equilibrium temperature caused by the curvature of the bubble surface. The radius R of this bubble coincides with that of the channel. By Eq. (10a), this temperature difference is just half of the difference $\Delta T = T_W - T_\infty$. Note that, although apparently invariant with respect to the co-ordinate x, Eq. (11) holds only at a particular axial position within the channel which can be determined by means of Eq. (3). Upstream of this position the wall temperature is too low to satisfy Eq. (7), whereas downstream, even smaller bubbles can survive there. Note, too, that Eq. (3) demands developed heat transfer conditions so that, in a general case, also the developing portion of the channel length needs to be considered.

Similarly, Eq. (6) that is valid for a bubble with $r_B = R/2$ and the equilibrium in the channel axis, gives

$$q_W=\frac{8}{3}\frac{2\sigma T_\infty}{h_{LV}\rho_V R}\frac{k_L}{R}, \tag{12}$$

which differs from Eq. (11) only by the numerical value of the constant.

Eq. (11) may be compared with the one reported by Peng et al. [12]. Setting $D_h = 2R$ and neglecting the specific volume of liquid in comparison with that of vapour, their expression can be written as

$$q_W \geqslant \frac{2}{c\pi}\frac{h_{LV}}{c_{pV}}\frac{k_V}{R}, \tag{13}$$

where k_V is the thermal conductivity, c_{pV} the specific heat capacity of vapour, and c is an empirical constant. The disagreement of this equation with Eq. (11) is caused by

[1] Clearly, in this case the bubble cannot be fixed in the channel under flow conditions. This hypothetical situation is taken for purposes of comparison with the literature [12].

different models adopted in the present paper and by Peng et al. [12].

Peng et al. [12] started from a stability criterion and arrived at

$$\left(\frac{\partial p}{\partial T}\right)_{\mathrm{S}} > \left(\frac{\partial S}{\partial V}\right)_{\mathrm{T}}. \tag{14}$$

The left-hand side in this inequality was obtained from the Clausius–Clapeyron equation, which requires a two-phase system, stating (see e.g. [20])

$$\left(\frac{\partial p}{\partial T}\right)_{\mathrm{V}} = \left(\frac{\partial S}{\partial V}\right)_{\mathrm{T}} = \frac{S_{\mathrm{V}} - S_{\mathrm{L}}}{V_{\mathrm{V}} - V_{\mathrm{L}}}, \tag{15}$$

the subscript V attached to $\partial p/\partial T$ referring to constant volume, whereas the right-hand side in (14) was estimated for a system containing only the vapour phase that is heated at a constant wall heat flux q_{W} and a fixed Fourier number, $Fo = a_{\mathrm{V}}t/R^2 = c\pi$. This, however, makes their derivation procedure inconsistent.

Prior to proceeding to discuss one of the possible reasons why formation of "visible" bubbles in saturated liquid flowing inside a micro-channel may be suppressed or shifted towards higher heat fluxes in comparison with macro-channels, we should emphasise that, within the model used in the present paper, there is no specific effect of (micro) channel size on the interaction between the bubble equilibrium and heat flux. Eq. (11) has the same shape as those valid under macro-channel flow conditions, see e.g. [9,11] for their experimental validation, and from this point of view, this equation is not new. It may be of interest that an expression deduced by Faneuff et al. [1] from an unsteady heat conduction model at unconfined flow conditions reduces, for a saturated liquid, practically to Eq. (11) when the cavity radius is set equal to the channel radius, see also a discussion provided by Zuber in [3].

3. Suppression of formation of large bubbles

Assuming, as before, the flow to be laminar and the pressure drop in the micro-channel to behave like in macro-channels, then, the wall shear stress τ_{W} of a single-phase flow becomes

$$\tau_{\mathrm{W}} = 4\frac{v_{\mathrm{L}}}{R}\dot{m}, \tag{16}$$

where v_{L} is the liquid kinematic viscosity and $\dot{m} = \rho_{\mathrm{L}}u$ is the mass flow density. Involving Newton's law, we get the vorticity $\mathrm{d}u/\mathrm{d}r$ of the flow field on the wall surface to be, approximately,

$$\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)_{\mathrm{W}} \sim \frac{\dot{m}}{\rho_{\mathrm{L}}R}. \tag{17}$$

Supposing a vapour bubble of a radius r_{B} to adhere to the wall, we may estimate a shear lift force F acting on the bubble as

$$F \sim V_{\mathrm{B}}\rho_{\mathrm{L}}u_{*}\left(\frac{\mathrm{d}u}{\mathrm{d}r}\right)_{\mathrm{W}} \sim r_{\mathrm{B}}^3\frac{u_{*}\dot{m}}{R} \sim \frac{\dot{m}^2}{R^2\rho_{\mathrm{L}}}r_{\mathrm{B}}^4, \tag{18}$$

where V_{B} represents the bubble volume and $u_{*} \approx 2ur_{\mathrm{B}}/R$ is the liquid velocity at the distance r_{B} from the wall. Expressions (16) and (18) suffice for our qualitative discussion so that other forces acting on the bubble, e.g., pressure drop and inertia forces, do not need to be considered here; the reader may be referred to the literature for details and further hydrodynamic effects, e.g. [21,22].

The shear stress (16) and the lift force (18) facilitate the bubble sliding and bubble detachment; these forces are larger for narrow channels. The force F sensitively increases as the bubble grows. Thus, when a vapour bubble growing on the wall surface of a micro-channel is swept away from its formation site at a radius smaller than the channel radius ($r_{\mathrm{B}} < R$), the hydrodynamic forces, and in the first place the lift force (18), tend to drive the bubble towards the channel axis. In the wall-far region of the cross-sectional flow area (liquid bulk), such a bubble may become unstable and condense, Fig. 1(b). For tiny bubbles, even a superheated liquid may appear as subcooled, resulting in a process that is analogous to boiling of an actually subcooled liquid. Given the mass flow density $\dot{m}$, the bubble detachment size is smaller at a smaller channel radius. This leads to a stronger suppression of formation of visible bubbles the narrower the flow channel.

In context with the bubble suppression discussed above, the papers by Ward et al. [23] and Brereton et al. [24], dealing with nucleation in confined volume and capillaries and both starting from dissolved gases, should be mentioned. Ward et al. showed theoretically that, for a stable gas bubble to develop within the liquid–gas solution, the content of the gas dissolved in the liquid must exceed a threshold value. Brereton et al. reported the nucleation temperature to increase as the capillary diameter decreases. A further paper, by Yuan et al. [25], is devoted to bubble dynamics in capillaries connecting two liquid reservoirs.

4. Conclusion

An equation is derived for the heat flux that must be applied if a vapour bubble should be able to survive in a micro-channel under the specified heat transfer conditions. This equation does not indicate any principal difference between micro- and macro-channel flow conditions with regard to the interaction between bubble equilibrium and heat flux. The suppression of formation of a visible vapour bubble, or its shifting towards higher heat fluxes, is discussed in terms of a premature bubble detachment caused by hydrodynamic forces. On the basis of simple expressions, it was concluded that the suppression effect is stronger the smaller the channel.

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